Lecture 19 - Nov. 14

Bridge Controller

Discharging Guard Strengthening POs
Invariant Preservation: Concrete Events
Commuting Diagram: Simulation
New Events: IL_in, IL_out

Announcements/Reminders

- **Lab5** to be released on by next Tuesday's class (Nov 19)
  (due on Tuesday, December 3)
- **WrittenTest2** next Wednesday, November 20
  + Guide
  + Practice Questions
- **Bonus** Opportunity coming: Formal Course Evaluation

Discharging POs of m1: Guard Strengthening in Refinement

ML_out/GRD

$$\frac{H1 \vdash G}{H1, H2 \vdash G}\ \text{MON} \qquad \frac{}{H, P \vdash P}\ \text{HYP}$$

$$\frac{H(F), E = F \vdash P(F)}{H(E), E = F \vdash P(E)}\ \text{EQ\_LR}$$

axioms
- $d \in \mathbb{N}$
- $d > 0$

abstract I
- $n \in \mathbb{N}$
- $n \leq d$

Con. Inv.
- $a \in \mathbb{N}$
- $b \in \mathbb{N}$
- $c \in \mathbb{N}$
- $a + b + c = n$
- $a = 0 \vee c = 0$

Con. guards of ML_out
- $a + b < d$
- $c = 0$

$\vdash$

abst. guard of ML_out
- $n < d$

MON → $a+b+c=n,\ a+b<d,\ c=0 \vdash n<d$

EQ_LR, MON → $a+b+0=n,\ a+b<d \vdash n<d$

$a+b+0 = a+b=n$ ↗ ARI

$a+b=n,\ a+b<d \vdash n<d$ (R)

EQ_LR, MON → $n<d \vdash n<d$

HYP

Discharging POs of m1: Guard Strengthening in Refinement

ML_in/GRD

$$d \in \mathbb{N}$$
$$d > 0$$
$$n \in \mathbb{N}$$
$$n \leq d$$
$$a \in \mathbb{N}$$
$$b \in \mathbb{N}$$
$$c \in \mathbb{N}$$
$$a + b + c = n$$
$$a = 0 \lor c = 0$$
$$c > 0$$
$$\vdash$$
$$n > 0$$

$$\frac{H1 \vdash G}{H1, H2 \vdash G}\ \text{MON} \qquad \frac{}{H, P \vdash P}\ \text{HYP} \qquad \frac{}{\bot \vdash P}\ \text{FALSE\_L}$$

$$\frac{H(F), E = F \vdash P(F)}{H(E), E = F \vdash P(E)}\ \text{EQ\_LR} \qquad \frac{H, P \vdash R \quad H, Q \vdash R}{H, P \lor Q \vdash R}\ \text{OR\_L}$$

ARI
$n>0$
$\vdash$
$n>0$
HYP

MON
$b \in \mathbb{N}$
$a+b+c=n$
$a=0 \lor c=0$
OR_L
$c>0$
$\vdash$
$n>0$

$b \in \mathbb{N}$
$a+b+c=n$
$a=0$
$c>0$
$\vdash$
$n>0$
EQ_LR, MON
$b \in \mathbb{N}$
$0+b+c=n$
$c>0$
$\vdash$
$n>0$
ARI
$b \in \mathbb{N}$
$b+c=n$
$c>0$
$\vdash$
$n>0$
ARI
$c \leq n$
$c>0$
$\vdash$
$n>0$
*

n
0 c

$b \in \mathbb{N}$
$a+b+c=n$
$c=0$
$c>0$
$\vdash$
$n>0$
MON
$c=0$
$c>0$
$\vdash$
$n>0$
$c=0 \land c>0 \equiv \bot$
ARI
$\bot$
$\vdash$
$n>0$
FALSE_L

for concrete evts

PO/VC Rule of Invariant Preservation: Sequents

Abstract m0 * $a'+b'+c'=n'$ $(a+1)+b+c=n+1$

variables: n

invariants:
- inv0_1 $n \in \mathbb{N}$
- inv0_2 $n \le d$

ML_out
when $n < d$
then $n := n+1$
end $n'=n+1$

ML_in
when $n > 0$
then $n := n-1$
end $n'=n-1$

$A(c)$ axioms
$I(c, v)$ abst. I
$J(c, v, w)$ conc. I.
$H(c, w)$ con. guard
$\vdash$
$J_i(c, E(c, v), F(c, w))$ $i \in 1..5$

Concrete m1 ** $a'=0 \vee c'=0$ $a=0 \vee c-1=0$

variables: a, b, c

invariants:
- inv1_1 $a \in \mathbb{N}$
- inv1_2 $b \in \mathbb{N}$
- inv1_3 $c \in \mathbb{N}$
- inv1_4 $a+b+c=n$
- inv1_5 $a=0 \vee c=0$

ML_out
when $a+b<d$, $c=0$
then $a := a+1$
end $a'=a+1$, $b'=b$, $c'=c$

ML_in
when $c > 0$
then $c := c-1$
end $c'=c-1 \wedge a'=a$, $b'=b$

Concrete invariant satisfied in the post state.

ML_out/inv1_4/INV
- $d \in \mathbb{N}$
- $d > 0$
- $n \in \mathbb{N}$
- $n \le d$
- $a \in \mathbb{N}$ $b \in \mathbb{N}$ $c \in \mathbb{N}$
- $a+b+c=n$
- $a=0 \vee c=0$
- $a+b<d$
- $c=0$
- $\vdash$
- * $(a+1)+b+c=n+1$

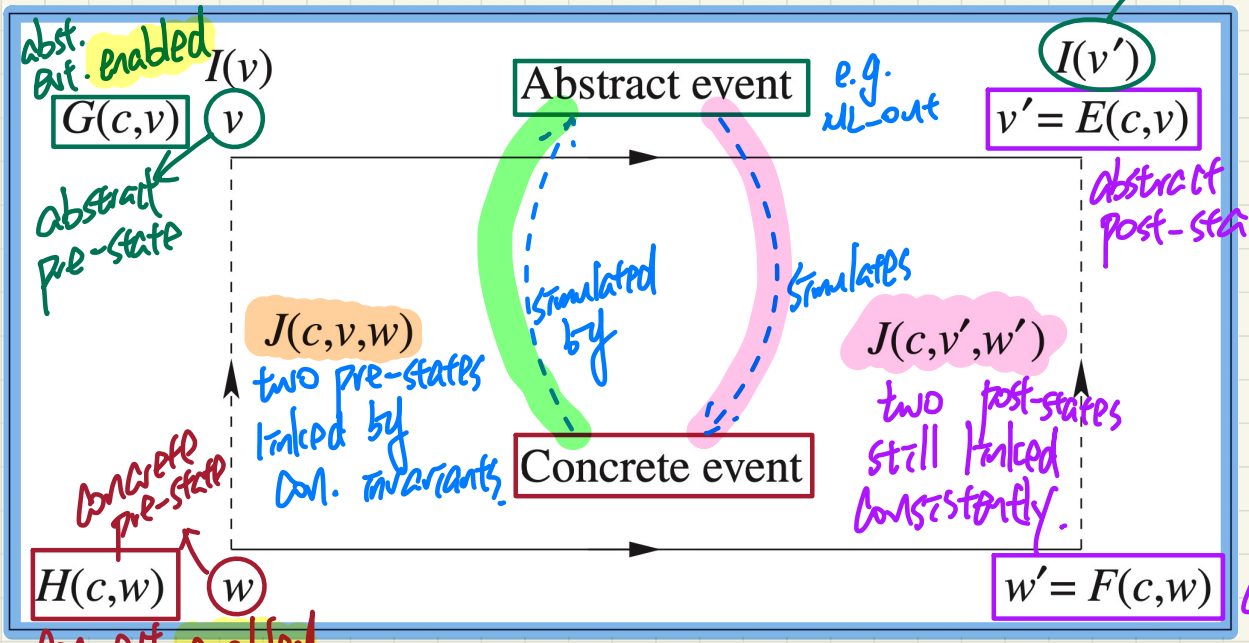
ML_in/inv1_5/INV
- $d \in \mathbb{N}$
- $d > 0$
- $n \in \mathbb{N}$
- $n \le d$
- $a \in \mathbb{N}$ $b \in \mathbb{N}$ $c \in \mathbb{N}$
- $a+b+c=n$
- $a=0 \vee c=0$
- $c>0$
- $\vdash$
- ** $a=0 \vee c-1=0$

2 con. evts * 5 inv. = 10

Q. How many PO/VC rules for model m1?

Visualizing Invariant Preservation in Refinement

Each **concrete** state transition (from w to w')
should be simulated by
an **abstract** state transition (from v to v')



Discharging POs of m1: Invariant Preservation in Refinement

ML_out/inv1_4/INV

$$d \in \mathbb{N}$$
$$d > 0$$
$$n \in \mathbb{N}$$
$$n \leq d$$
$$a \in \mathbb{N}$$
$$b \in \mathbb{N}$$
$$c \in \mathbb{N}$$
$$a + b + c = n$$
$$a = 0 \lor c = 0$$
$$a + b < d$$
$$c = 0$$
$$\vdash$$
$$(a + 1) + b + c = (n + 1)$$

$$\frac{H1 \vdash G}{H1, H2 \vdash G}\ \text{MON}$$

$$\frac{}{P \vdash E = E}\ \text{EQ}$$

$$\frac{H(F), E = F \vdash P(F)}{H(E), E = F \vdash P(E)}\ \text{EQ\_LR}$$

Discharging POs of m1: Invariant Preservation in Refinement

ML_in/inv1_5/INV

$$d \in \mathbb{N}$$
$$d > 0$$
$$n \in \mathbb{N}$$
$$n \leq d$$
$$a \in \mathbb{N}$$
$$b \in \mathbb{N}$$
$$c \in \mathbb{N}$$
$$a + b + c = n$$
$$a = 0 \lor c = 0$$
$$c > 0$$
$$\vdash$$
$$a = 0 \lor (c - 1) = 0$$

$$\frac{}{H, P \vdash P}\ \text{HYP}$$

$$\frac{}{\bot \vdash P}\ \text{FALSE\_L}$$

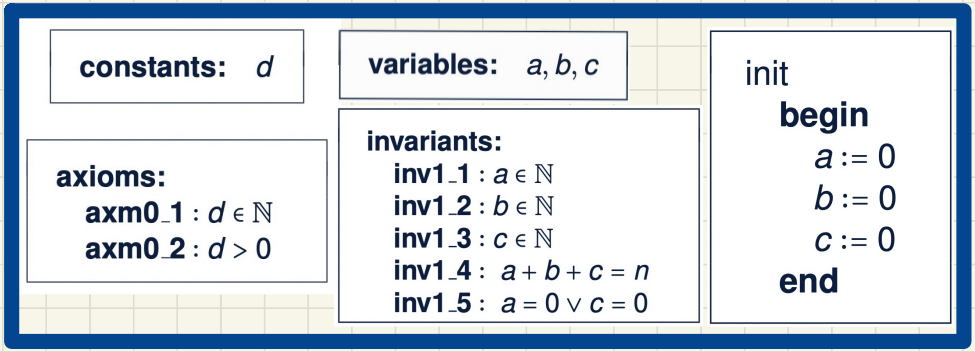
$$\frac{H1 \vdash G}{H1, H2 \vdash G}\ \text{MON}$$

$$\frac{H \vdash P}{H \vdash P \lor Q}\ \text{OR\_R1}$$

$$\frac{H(F), E = F \vdash P(F)}{H(E), E = F \vdash P(E)}\ \text{EQ\_LR}$$

$$\frac{H, P \vdash R \qquad H, Q \vdash R}{H, P \lor Q \vdash R}\ \text{OR\_L}$$

PO of Invariant Establishment in Refinement



Components

K(c): effect of abstract init

L(c): effect of concrete init

Rule of Invariant Establishment

$$A(c)$$
$$\vdash$$
$$J_i(c, K(c), L(c))$$

Exercise:

Generate Sequents from the INV rule.

Q. How many PO/VC rules for model m1?

Discharging PO of Invariant Establishment in Refinement

$$d \in \mathbb{N}$$
$$d > 0$$
$$\vdash$$
$$0 + 0 + 0 = 0$$

init/inv1_4/INV

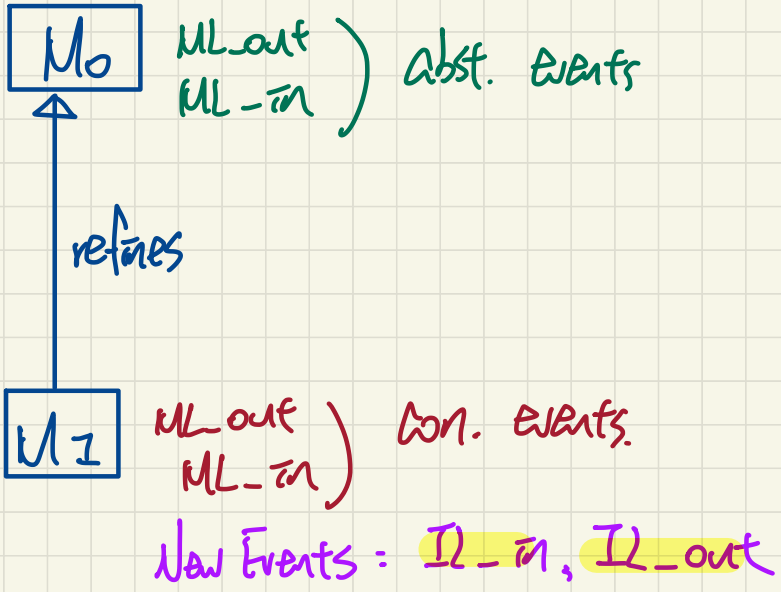
$$\frac{H1 \vdash G}{H1, H2 \vdash G}\ \text{MON}$$

$$\frac{}{P \vdash \top}\ \text{TRUE\_R}$$

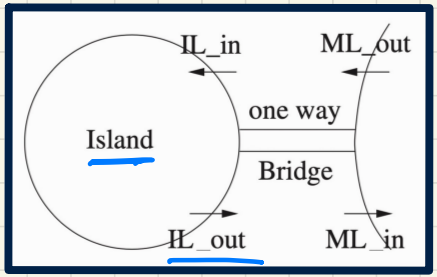
$$d \in \mathbb{N}$$
$$d > 0$$
$$\vdash$$
$$0 = 0 \lor 0 = 0$$

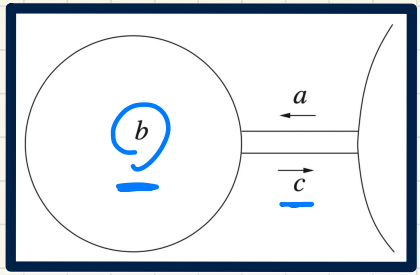
init/inv1_5/INV

Events


M0
ML_out
ML_in
abst. events
refines
M1
ML_out
ML_in
con. events
New Events : IL_in, IL_out

Bridge Controller: Guarded Actions of "new" Events in 1st Refinement





constants: d

axioms:
axm0_1 : $d \in \mathbb{N}$
axm0_2 : $d > 0$

variables: a, b, c

invariants:
inv1_1 : $a \in \mathbb{N}$
inv1_2 : $b \in \mathbb{N}$
inv1_3 : $c \in \mathbb{N}$
inv1_4 : $a + b + c = n$
inv1_5 : $a = 0 \vee c = 0$

IL_in: A car enters **island** (getting off the **bridge**).

IL_in
when
??
then
??
end

① $a > 0$
② $c = 0$

$a := a - 1$
$b := b + 1$

IL_out: A car exits **island** (getting on the **bridge**).

IL_out
when
??
then
??
end

① $b > 0$
② $a = 0$

$b := b - 1$
$c := c + 1$

Before-After Predicates of Event Actions: 1st Refinement

IL_in

when

$a > 0$

then

$a := a - 1$

$b := b + 1$

end

IL_out

when

$b > 0$

$a = 0$

then

$b := b - 1$

$c := c + 1$

end

$a' = a - 1$

$\wedge\ b' = b + 1$

$\wedge\ c' = c$

- Pre-State
- Post-State
- Sate Transition

Concrete State Space